Mathematics (SET)

1. For sets A, B, C and D, he equation
A X B − C X D = (A−C) X (B−D) is
(**A**) True if $C \subseteq A$ & $D \subseteq B$
(**B**) False if $C \subseteq A$ & $D \subseteq B$
(**C**) Always true (**D**) None of these

2. Let $f : A \to B$ be a function
and let $f^{-1}(B) = \{x : f(x) \in B\}$.
Then choose the correct statement.
(**A**) $f(f^{-1}(B)) = B$ (**B**) $f^{-1}(f(A)) = A$
(**C**) $B \subseteq f(f^{-1}(B))$ (**D**) $f(f^{-1}(B)) \subseteq B$

3. $f : R \to R$ given by $f(x) = 2x + \sin x$ is
(**A**) 1-1 & not onto (**B**) onto & not 1-1
(**C**) 1-1 & onto (**D**) neither 1-1 nor onto

4. How many of the functions $|x|$, $|x|^2$, $|x|^3$ and $|x|^5$ are not differentiable at 0 ?
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4

5. Let $f : X \to Y$ be a continuous function where X and Y are metric spaces and if $E \subseteq X$, then
(**A**) $f(\overline{E}) \subseteq \overline{f(E)}$ (**B**) $f(\overline{E}) = \overline{f(E)}$
(**C**) $\overline{f(E)} \subseteq f(\overline{E})$ (**D**) $\overline{f(E)} \subset f(\overline{E})$

6. Choose the wrong statement.
(**A**) It is possible that the countable intersection of open sets is an open set.
(**B**) It is possible that the countable union of closed sets is a closed set.
(**C**) The countable union of open sets is always an open set.
(**D**) The countable intersection of open sets is never an open set.

7. For a set A, if the closure is denoted by $\overline{A}$, then which statement is always true ?
(**A**) $\overline{A \cap B} = \overline{A} \cap \overline{B}$
(**B**) $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$
(**C**) $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$
(**D**) $\overline{A \cap B} \supseteq \overline{A} \cap \overline{B}$

8. Consider the set A = {1, 2, 3, 4} as a subset of Real Line R with usual topology $\mathcal{T}$. Among the following statements, which all can be used to observe that A is a closed set ?
(i) Any finite set in $(R, \mathcal{T})$ is closed.
(ii) A contains all its limit points.
(iii) The complement of A is an open set.
(**A**) (i) & (ii) (**B**) (ii) & (iii) (**C**) (i) & (iii)
(**D**) All of them.

9. Let $f : R \to R$ be defined as
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0. \end{cases}$$ Then
(**A**) f is continuous and differentiable everywhere
(**B**) f is continuous everywhere and differentiable at non-zero real numbers
(**C**) f is neither continuous nor differentiable at real points
(**D**) f is continuous and differentiable at all non-zero points

10. The number of real roots of $\frac{x}{100} = \sin x$ is
A) 61 (**B**) 62 (**C**) 63 (**D**) 64o points
(**C**) f is neither continuous nor differentiable
(**D**) f is continuous and differentiable only at non-zero points

11. Numbers can be rational, irrational, algebraic or transcendental.
How many properties the number π has ?
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3

12. The number of real roots of $\frac{x}{100} = \sin x$ is
A) 61 (**B**) 62 (**C**) 63 (**D**) 64

13. The number of real roots of $\log x = x$ is
A) 0 (**B**) 1 (**C**) 2 (**D**) Data insufficient

14. Let $f : R \to R$ be defined as
$$f(x) = \begin{cases} x & \text{if x is rational} \\ 0 & \text{if x is irrational.} \end{cases}$$ Then
(**A**) f is continuous at all rationals
(**B**) f is continuous at all irrationals
(**C**) f is continuous at x = 0 and has a discontinuity of 1^{st} kind at other points
(**D**) f is continuous at x = 0 and has a discontinuity of 2^{nd} kind at other points

15. Let X and Y be topological spaces.
Is it possible to have continuity for any function $f : X \to Y$ by choosing suitable topologies on X and Y ?

16. Let X and Y be topological spaces.
Is it possible that no function $f : X \to Y$, other

than the identity function, be continuous by choosing suitable topologies on X and Y ?

17. Let $f, g : R \to R$ be defined by $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ and $g(x) = \dfrac{1}{\sqrt{x - |x|}}$. Then
(A) $dom\, f = dom\, g$ (B) $dom\, f = \phi, dom\, g \neq \phi$
(C) $dom\, f \neq \phi, dom\, g \neq \phi$
(D) $dom\, f \neq \phi, dom\, g = \phi$

18. Domain of the function $f(x) = \sin^{-1}(\log_3 x)$ is
(A) $[0, 3]$ (B) $[\frac{1}{3}, 1]$ (C) $[\frac{1}{3}, 3]$ (D) $[1, 3]$

19. $f(x) = \log \left[\dfrac{1+x}{1-x}\right]$ satisfies the equation
(A) $f(x+2) - 2f(x+1) - f(x) = 0$
(B) $f(x) + f(x+1) = f(x(x+1))$
(C) $f(x_1)f(x_2) = f(x_1 + x_2)$
(D) $f(x_1) + f(x_2) = f\left(\dfrac{x_1 + x_2}{1 + x_1 x_2}\right)$

20. Let $\{x_n\}$ and $\{y_n\}$ sequences such that $\{x_n + y_n\}$ is convergent. Which of the following is the most suitable statement ?
(A) Both $\{x_n\}$ and $\{y_n\}$ are convergent sequences
(B) At least one of $\{x_n\}$ and $\{y_n\}$ will be a convergent sequence
(C) Neither $\{x_n\}$ nor $\{y_n\}$ are convergent sequences
(D) None of these.

21. Let $\{x_n\}$ and $\{y_n\}$ be convergent sequences. Then choose the most correct statement.
(A) $\{x_n + y_n\}$ is a convergent sequence
(B) $\{x_n - y_n\}$ is a convergent sequence
(C) Both $\{x_n + y_n\}$ and $\{x_n - y_n\}$ are convergent sequences
(D) Nothing can be said about the convergence of the sequences $\{x_n + y_n\}$ and $\{x_n - y_n\}$.

22. Let $f, g : R \to R$ be bounded functions and let $A, B \subseteq R$.
Let $\mathbf{p} = \sup\{x : (f+g)(x), x \in A\}$
$\mathbf{q} = \sup\{x : (f)(x), x \in A\}$ and
$\mathbf{r} = \sup\{x : (g)(x), x \in A\}$. Then
(A) $\mathbf{p} \leq \mathbf{q} + \mathbf{r}$
(B) $\mathbf{p} \geq \mathbf{q} + \mathbf{r}$
(C) $\mathbf{p} = \mathbf{q} + \mathbf{r}$
(D) $\mathbf{p}$ can not be compared with $\mathbf{q} + \mathbf{r}$.

23. Let $f, g : R \to R$ be bounded functions and let $A, B \subseteq R$.
Let $\mathbf{p} = \inf\{x : (f+g)(x), x \in A\}$
$\mathbf{q} = \inf\{x : (f)(x), x \in A\}$ and
$\mathbf{r} = \inf\{x : (g)(x), x \in A\}$. Then
(A) $\mathbf{p} \leq \mathbf{q} + \mathbf{r}$
(B) $\mathbf{p} \geq \mathbf{q} + \mathbf{r}$
(C) $\mathbf{p} = \mathbf{q} + \mathbf{r}$
(D) $\mathbf{p}$ can not be compared with $\mathbf{q} + \mathbf{r}$.

24. For $x \in R$, let $f_n(x) = \dfrac{x^{2n}}{1 + x^{2n}}$, $n \in N$ and let

$$f(x) = \lim_{n\to\infty} f_n(x).$$

Choose the correct statement about f.
(A) f is conitinuous in (-2, 0)
(B) f is conitinuous in (-1, 2)
(C) f is conitinuous in (1, 2)
(D) f is conitinuous in (0, 2)

25. For $n = 1, 2, 3, \cdots$, define

$$f_n(x) = \begin{cases} n^2 x & x \in (\frac{1}{n}, \frac{2}{n}) \\ 0 & \text{otherwise in } [0, 1]. \end{cases}$$

Then which of the following is true ?
(A)

$$\int_0^1 f_n(x)dx < 2 \quad \forall n$$

(B)

$$\int_0^1 f_n(x)dx > 2 \quad \forall n$$

(C)

$$\int_0^1 f_n(x)dx = \frac{5}{2} \text{ where } f(x) = \lim_{n\to\infty} f_n(x)$$

(D)

$$\int_0^1 f_n(x)dx = 1 \text{ where } f(x) = \lim_{n\to\infty} f_n(x)$$

26. $\lim_{x\to\infty} x^{2011}e^{-x}$ is
(A) 0
(B) 1
(C) 2011
(D) ∞

27. Let [a, b] and [c, d] be two intervals. Then
(A) Any function from one interval to the other is continuous.
(B) Any function from one interval to the other is invertible.
(C) There exists a function from one to the other such that it is 1-1 and onto and both the function and it's inverse are continuous.
(D) It is impossible to find a continuous function as mentioned in (C).

28. Let

$$f(x) = \begin{cases} 0 & \text{if x is irrational} \\ \frac{1}{q} & \text{if x is rational and } = \frac{p}{q}. \end{cases}$$

Then $\lim_{x \to \frac{1}{2}} f(x) =$
(A) 0 (B) $\frac{1}{2}$ (C) 2 (D) Limit does not exist.

29. For the function in the previous question, $\lim_{x \to \sqrt{2}} f(x) =$
(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) 0 (D) Limit does not exist.

30. If $\int_0^x f(t)dt = x + \int_x^1 f(t)dt$ then $f(1) =$
(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) 1 (D) -1

31.

$$\lim_{n \to \infty} \{\sqrt{(n^2+n)} - n\} \text{ is equal to}$$

(A) 0 (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) ∞

32.

$$\lim_{n \to \infty} \{\sqrt{(n^2+n)} - n\} \text{ is equal to}$$

(A) 0 (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) ∞

33.

$$\int_{-3}^{7} |x+1| dx =$$

(A) 30 (B) 31 (C) 32 (D) 34

34. If [x] is the greatest integer function,

$$\int_{-7}^{17} (x - [x]) dx =$$

(A) 10 (B) 11 (C) 12 (D) None of these

35. Let $f, g : R \to R$ be functions.
Let A = {x : f(x) = 0}
and B = {x : g(x) = 0}. Then A ∩ B =
(A) {x : $f(x) \times g(x) = 0$}
(B) {x : $f(x) + g(x) = 0$}
(C) {x : $f(x)^2 + g(x)^2 = 0$}
(D) {x : $f(x) = 0 = g(x)$}

36. Let $f, \dot{g} : R \to R$ be functions.
Let A = {x : f(x) = 0}
and B = {x : g(x) = 0}. Then A ∪ B =
(A) {x : $f(x) \times g(x) = 0$}
(B) {x : $f(x) + g(x) = 0$}
(C) {x : $f(x)^2 + g(x)^2 = 0$}
(D) {x : $f(x) = 0 = g(x)$}

37. If

$$p = \lim_{x \to 0} x \sin\frac{1}{x} \text{ and } q = \lim_{x \to \infty} x \sin\frac{1}{x},$$

then
(A) $p = 0$ & $q = 0$
(B) $p = 0$ & $q = \infty$
(C) $p = 0$ & $q = 1$
(D) $p = 1$ & $q = \infty$

38. Consider the sequence $\{x_n\}$ where $x_1 = 1$, $x_2 = 2$ and $x_{n+2} = x_{n+1} + x_n \forall n \in N$. Then
(A) The sequence $\{x_n\}$ is convergent to $\frac{5}{3}$
(B) The sequence $\{x_n\}$ is divergent
(C) The sequence $\{x_n\}$ is convergent to $\frac{9}{5}$
(D) The sequence $\{x_n\}$ is convergent to some other real number.

39. Consider the sequence $\{x_n\}$ where $x_1 = 1$ and $x_{n+1} = \sqrt{2 + x_n} \forall n \in N$. Then
(A) The sequence $\{x_n\}$ is convergent to $\sqrt{2}$
(B) The sequence $\{x_n\}$ is convergent to $\sqrt{3}$
(C) The sequence $\{x_n\}$ is convergent to 2
(D) The sequence $\{x_n\}$ is divergent.

40. Let A = $\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\} \subseteq R$.
Let us consider the properties
(i)Closedness, (ii)Compactness, (iii)Boundedness.
Then A satisfies
(A) (i) and (iii)
(B) (ii) and (iii)
(C) only (iii)
(D) (i) , (ii) and (iii)

41. The limit of the series $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\cdots=$
(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) None of these

42. Choose the divergent series.

(A) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

(B) $\sum_{n=1}^{\infty}\frac{1}{n\log(n+1)}$

(C) $\sum_{n=1}^{\infty}\frac{\sqrt{n-1}-\sqrt{n}}{n}$

(D) $\sum_{n=1}^{\infty}\frac{1}{n^2}$

43. Which of the following sequence is convergent for all x in [0, 1], but not uniformly convergent on [0, 1] ?
(A) $\left\{\frac{\sin nx}{\sqrt{n}}\right\}$
(B) $\left\{\sin nx\right\}$
(C) $\left\{x^n(1+x)^{-n}\right\}$
(D) $\left\{x^n\right\}$

44. If $\{\{1, 2\}, \{3, 4, 5, 6\}, \{7, 8\}\}$ is a partition of the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ then what is the number of ordered pairs in the equivalence relation on A corresponding to the partition ?
(A) 8 (B) 16 (C) 24 (D) 32

45. If $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{pmatrix}$ then $A^{101}=$
(A) I (B) $A-I$ (C) A (D) $(a+b)(A-I)$

46.
$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=$$
(A) 1 (B) 0 (C) e (D) ∞

47.
$$\lim_{n\to\infty}\left(n^{\frac{1}{n}}-1\right)^n=$$
(A) 1 (B) 0 (C) e (D) ∞

48.
$$\lim_{x\to 0}\left(\cos x\right)^{\cot x}=$$
(A) 1 (B) 0 (C) e (D) ∞

49. If $f(x)=x-x^2+x^3+\cdots to\,\infty$ for $|x|<1$, then $f^{-1}(x)=$
(A) x (B) $\frac{x}{1-x}$ (C) $\frac{1-x}{x}$ (D) $\frac{x}{1+x}$

50. What is the value of x for which $\sum_{n=1}^{20}(x-n)^2$ is the least ?
(A) 1 (B) 10 (C) 20 (D) None

51. How many real roots the quadratic equation $(x-2)^2+(x-5)^2+(x-7)^2=0$ has ?
(A) 1 (B) 2 (C) 3 (D) None

52. If x satisfies the equation $x^2-2\cos x+1=0$, then the value of $x^n+\frac{1}{x^n}$ is
(A) $2\cos nx$ (B) $2^n\cos nx$
(C) $2\cos^n x$ (D) $2^n\cos^n x$

53. The spheres $x^2+y^2+z^2=25$ and $x^2+y^2+z^2-24x-40y-18z+225=0$
(A) touch internally
(B) touch externally
(C) do not touch each other
(D) intersect each other

54. If $f(x)=\min\{x, x^2\}$ for every real value of x, then which one of the following is not true ?
(A) f is continuous for all x
(B) f is differentiable for all x
(C) $f'(x)=1$ for all x
(D) one of the above statement is wrong

55. Consider the intervals
A = [0, 1], B = [0, 1), C = (0, 1] and D = (0, 1) as subsets of R with usual topology.
Between which two sets it is possible to have a homeomorphism ? Give reason for your answer.

56. If z satisfies $\left|z+\frac{1}{z}\right|=4$, then the maximum value of $|z|$ is
(A) $2-\sqrt{5}$ (B) $2+\sqrt{5}$
(C) $4-\sqrt{5}$ (D) $4+\sqrt{5}$

57. The sum ${}^{100}C_0+{}^{101}C_1+{}^{102}C_2\cdots+{}^{150}C_{50}=$
(A) ${}^{200}C_{100}$ (B) ${}^{201}C_{50}$
(C) ${}^{201}C_{100}$ (D) ${}^{151}C_{50}$